Special Relativity Regime Compiler (SRRC)

A Reduced Instruction Set for Relativistic Invariants

Adrian Diamond

2026

Contents

Abstract

Special Relativity is characterized by invariance under the Lorentz group. This paper isolates its invariant algebra by quotienting observational space under Lorentz action and identifying regime

space as the orbit space. In the single-event case, the invariant algebra is generated by the Minkowski quadratic form. The Special Relativity Regime Compiler (SRRC) is the categorical factorization of invariant observables through the orbit projection, rendering the invariant content of the theory as an explicitly generated invariant algebra. All constructions are real-analytic and defined over $\mathbb{R}$.

1 Motivation and Scope

This paper extracts the invariant algebra implicit in Special Relativity. The objective is not reinterpretation, but structural reduction to Lorentz-invariant primitives.

Observational structure is reduced via the orbit projection

$$\pi : \mathbb{R}^{1,3} \to \mathbb{R}^{1,3}/O(1,3),$$

and invariant observables are characterized as functions that factor through this quotient.

2 Regime Space in Special Relativity

Special Relativity defines equivalence classes of observational outcomes under Lorentz transformation.

Definition 2.1 (Relativistic Regime)**.** A relativistic regime is a Lorentz orbit

$$[x] = \{\Lambda x \mid \Lambda \in O(1,3)\}.$$

The regime space is the orbit space

$$\mathcal{R} = \mathbb{R}^{1,3}/O(1,3).$$

Remark 2.2 (Action Groupoid). The Lorentz action defines an action groupoid

$$O(1,3) \ltimes \mathbb{R}^{1,3},$$

whose objects are events and whose morphisms are Lorentz transformations. The regime space is the coarse moduli space of this groupoid.

Proposition 2.3 (Coarse Moduli Space)**.** *The orbit space $\mathcal{R}$ is the coarse moduli space of the action groupoid $O(1,3) \ltimes \mathbb{R}^{1,3}$: it corepresents invariant functions.*

Equivalently, continuous (or polynomial) functions constant on orbits factor uniquely through π.

Remark 2.4 (Reductive Structure and Base Field). The Lorentz group $O(1,3)$ is a real reductive Lie group. All constructions in this paper are carried out over $\mathbb{R}$. When applying classical invariant theory results, one may pass to the complexification $O(4,\mathbb{C})$; the invariant polynomial structure descends to the real form.

Proposition 2.5 (Universal Property of the Orbit Quotient)**.** *Let $f : \mathbb{R}^{1,3} \to \mathbb{R}$ be Lorentz-invariant. Then there exists a unique map*

$$\overline{f} : \mathcal{R} \to \mathbb{R}$$

such that

$$f = \overline{f} \circ \pi.$$

Regimes are orbit equivalence classes, not interpretive constructs.

3 Equivalence Classes and Invariants

3.1 Minkowski Space and Lorentz Action

Let $\mathbb{R}^{1,3}$ denote Minkowski space equipped with the bilinear form

$$\eta(x,x) = -t^2 + x_1^2 + x_2^2 + x_3^2,$$

where $x = (t, x_1, x_2, x_3)$.

The Lorentz group is defined as

$$O(1,3) = \{\Lambda \in GL(4,\mathbb{R}) \mid \eta(\Lambda x, \Lambda y) = \eta(x,y)\}.$$

The group acts on $\mathbb{R}^{1,3}$ by linear transformation:

$$x \mapsto \Lambda x.$$

Definition 3.1 (Observational Equivalence)**.** Two observational outcomes are equivalent if they are related by a Lorentz transformation and yield identical invariant predictions.

Formally, define the equivalence relation

$$x \sim y \quad \text{iff} \quad \exists \Lambda \in O(1,3) \text{ such that } y = \Lambda x.$$

The space of relativistic regimes is therefore the orbit space

$$\mathcal{R} = \mathbb{R}^{1,3}/O(1,3).$$

An invariant observable is a function

$$f : \mathbb{R}^{1,3} \to \mathbb{R}$$

satisfying

$$f(\Lambda x) = f(x) \quad \text{for all } \Lambda \in O(1,3).$$

The collection of all such invariant functions forms an algebra, denoted

$$\mathcal{A}^{O(1,3)},$$

the invariant algebra under the Lorentz action.

Lemma 3.2. *Only Lorentz-invariant quantities descend to well-defined functions on regime space.*

Proof. Lorentz invariance is precisely the condition required for well-definedness on the quotient space of observational equivalence classes. □

Proposition 3.3. *The Minkowski norm $\eta(x, x)$ is Lorentz invariant.*

Proof. By definition of $O(1, 3)$,

$$\eta(\Lambda x, \Lambda x) = \eta(x, x).$$

□

4 The Invariant Relativity Kernel

Let $V = \mathbb{R}^{1,3}$.

The invariant algebra is

$$\mathcal{A}^{O(1,3)} = \mathbb{R}[V]^{O(1,3)},$$

the algebra of polynomial functions invariant under the Lorentz action.

In this paper we restrict to polynomial invariants; smooth invariant functions factor through the same generators.

Theorem 4.1 (Single-Event Invariant Generation)**.** *In the single-event case, the invariant algebra*

$$\mathbb{R}[V]^{O(1,3)}$$

is generated by the Minkowski quadratic form

$$Q(x) = \eta(x, x).$$

Proof. The Lorentz group preserves η by definition. Classical invariant theory for the orthogonal group implies that the invariant polynomial algebra is generated by the quadratic form. $\square$

Thus the invariant kernel in the single-event case reduces to one generator.

5 Opcode Structure

In the single-event case, the invariant kernel admits a single generator:

$$\Omega(x) = \eta(x, x).$$

Definition 5.1 (Opcode)**.** An opcode is a morphism

$$\omega : \mathcal{R} \to \mathbb{R}$$

induced by an invariant polynomial on V.

In the single-event regime, the minimal opcode set consists of

$$\{\Omega\}.$$

All invariant observables are polynomial expressions in Ω.

6 Factorization Architecture

Let $V = \mathbb{R}^{1,3}$ and $\mathcal{R} = V/O(1,3)$.

Invariant observables factor uniquely through the orbit projection:

$$\begin{array}{ccc} V & \xrightarrow{\pi} & \mathcal{R} \\ & {\scriptstyle f}\searrow & \downarrow{\scriptstyle \overline{f}} \\ & & \mathbb{R} \end{array}$$

Every Lorentz-invariant observable satisfies

$$f = \overline{f} \circ \pi.$$

This defines a functor from the action groupoid

$$O(1,3) \ltimes V$$

to the category of real algebras, determined by invariant polynomial functions.

7 Invariant Factorization and Completeness

Theorem 7.1 (Operational Completeness)**.** *Every observer-independent polynomial observable is a polynomial in the Minkowski quadratic form.*

Proof. By the fundamental theorem of invariant theory for orthogonal groups, the invariant algebra is generated by the quadratic form. □

8 Multi-Event Regimes and Gram Matrix Invariants

Let $V = \mathbb{R}^{1,3}$ and consider k events:

$$(x_1, \ldots, x_k) \in V^k.$$

The Lorentz group acts diagonally:

$$\Lambda \cdot (x_1, \ldots, x_k) = (\Lambda x_1, \ldots, \Lambda x_k).$$

Definition 8.1 (Gram Matrix)**.** The Gram matrix associated to a k-tuple is

$$G_{ij} = \eta(x_i, x_j).$$

Each G_{ij} is Lorentz-invariant.

Theorem 8.2 (Invariant Generation — Multi-Event Case)**.** *The polynomial invariant algebra*

$$\mathbb{R}[V^k]^{O(1,3)}$$

is generated by the Gram matrix entries

$$\{G_{ij}\}_{1 \le i \le j \le k}.$$

Proof. This follows from the First Fundamental Theorem for orthogonal groups. □

Theorem 8.3 (Relations Among Invariants)**.** *The relations among the generators $\{G_{ij}\}$ are generated by determinantal identities corresponding to rank constraints imposed by the signature* $(1,3)$*.*

Proof. This follows from the Second Fundamental Theorem for orthogonal groups, which identifies the ideal of relations among Gram invariants. □

Corollary 8.4. *The invariant algebra admits the presentation*

$$\mathbb{R}[V^k]^{O(1,3)} = \mathbb{R}[G_{ij}]/I_{rank},$$

where I_{rank} is the ideal generated by signature-induced determinantal relations.

This identifies the invariant kernel in the multi-event case with the classical invariant theory of orthogonal groups. The First Fundamental Theorem (FFT) provides generators, while the Second Fundamental Theorem (SFT) identifies the ideal of relations.

In particular, I_{rank} contains all (5×5) minors of (G_{ij}), reflecting $\dim V = 4$.

9 Orbit-Type Stratification

The orbit space $\mathcal{R}$ is stratified by causal type: timelike, spacelike, and null regimes.

Proposition 9.1. *Orbit-type strata correspond to conjugacy classes of stabilizer subgroups in $O(1,3)$.*

Proof. This follows from orbit-type stratification theory for real-analytic Lie group actions, applied to the Lorentz action on Minkowski space. □

In the single-event case, null orbits possess larger stabilizers and form a singular stratum. Concretely, a null stabilizer contains a subgroup isomorphic to the Euclidean motion group $E(2)$.

Theorem 9.2. *The orbit space $\mathcal{R}$ admits the structure of a Whitney stratified space.*

Proof. For finite-dimensional real-analytic group actions, orbit-type stratifications satisfy Whitney conditions.[1] □

Although orbit spaces of non-compact groups may exhibit non-Hausdorff or singular behavior in general, the finite-dimensional real-analytic setting ensures controlled stratified structure in this case.

10 Conclusion

Special Relativity admits reduction to an invariant algebra determined by the Lorentz action. The regime space is the coarse moduli space of this action, stratified by orbit type. Single-event

[1] See standard slice results for real-analytic Lie group actions and orbit-type decompositions in transformation group theory.

invariants reduce to one quadratic generator, while multi-event invariants form a polynomial algebra modulo signature constraints. The factorization architecture makes invariant structure explicit without altering the underlying physical theory.

A Appendix: References

References

[1] A. Einstein, *On the Electrodynamics of Moving Bodies*, Annalen der Physik, 1905.

[2] H. Minkowski, *Space and Time*, 1908.

[3] R. M. Wald, *General Relativity*, University of Chicago Press, 1984.

[4] H. Weyl, *The Classical Groups*, Princeton University Press, 1939.

[5] C. Procesi, *Lie Groups: An Approach through Invariants and Representations*, Springer, 2007.

[6] S. Mac Lane, *Categories for the Working Mathematician*, Springer, 1998.