String-Theoretic Models of Semantic Cohomology

Worldsheets, Gauge Fields, and Topological Obstructions to Meaning

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Abstract

We develop a string-theoretic model of semantic structure in which local interpretations are connected by transition data analogous to strings. In this framework, semantic regimes are not globally defined objects, but arise from the coherent gluing of local structures across overlapping domains.

We show that strings naturally encode transition functions between local semantic states, while failures of global coherence correspond to nontrivial cohomological obstruction classes. This establishes a direct correspondence between string-theoretic structure, higher gauge theory, and semantic descent.

Event horizons and related physical structures are thereby interpreted as regions where semantic gluing fails, and where nontrivial higher-order structure emerges. This provides a unified geometric framework linking string theory, cohomology, and the structure of meaning.

Contents

1 Introduction

We develop a string-theoretic model of semantic structure in which meaning is governed by the interaction of local state spaces, transformation laws, and global consistency conditions.

The central principle is that semantic systems are not static collections of interpretations, but dynamical systems defined by admissible transformations acting on a space of states. Meaning is identified with invariant structure under these transformations, while failures of global consistency arise as topological obstructions.

In this framework, local semantic regimes are modeled as worldsheet-like domains supporting structured observational data. Compatibility between regimes is governed by transition functions analogous to gauge fields, and global semantic coherence corresponds to the existence of a globally trivial configuration.

We show that failures of semantic descent correspond to nontrivial cohomology classes, and that boundaries such as event horizons may be interpreted as obstructions analogous to branes in string theory.

This establishes a structural correspondence between semantic systems, gauge theory, and string-theoretic topology, in which meaning behaves as a global consistency condition on a system of locally defined states.

2 State Space and Transformation Law

We begin by formalizing the fundamental components of a semantic system: states and transformations.

2.1 Semantic State Space

Let M be a base space of observation. A semantic system assigns to each open set $U \subset M$ a set of states $\mathcal{O}(U)$, representing observational or representational configurations.

[Semantic State] A semantic state on U is an element

$$x \in \mathcal{O}(U),$$

representing a configuration of observational data over the domain U.

The assignment

$$U \mapsto \mathcal{O}(U)$$

defines a presheaf of state spaces over M.

2.2 Transformation Law (Action)

Semantic structure is determined not only by states, but by the transformations acting on them.

[Semantic Transformation] For each open set $U \subset M$, let $\mathcal{G}(U)$ denote a collection of admissible transformations acting on $\mathcal{O}(U)$. A transformation

$$g \in \mathcal{G}(U)$$

acts on a state $x \in \mathcal{O}(U)$ to produce a new state

$$g \cdot x \in \mathcal{O}(U).$$

Thus, $\mathcal{G}(U)$ defines an action on $\mathcal{O}(U)$.

[Semantic Action] A semantic action is the assignment

$$\mathcal{G}(U)\mathcal{O}(U)$$

for each open set U, specifying how states evolve under admissible transformations.

The action encodes the fundamental dynamics of the semantic system: it determines which state changes are allowed and how different representations are related.

2.3 Equivalence and Invariance

Transformations induce equivalence relations on states.

[Semantic Equivalence] Two states $x, y \in \mathcal{O}(U)$ are equivalent if there exists a transformation $g \in \mathcal{G}(U)$ such that

$$y = g \cdot x.$$

The quotient

$$\mathcal{R}(U) := \mathcal{O}(U)/\mathcal{G}(U)$$

defines the invariant semantic content over U.

[Semantic Invariant] A semantic invariant is a quantity that is constant on equivalence classes in $\mathcal{R}(U)$.

Thus, meaning is identified with invariance under the semantic action.

2.4 Interpretation as Worldsheet Dynamics

The structure $(\mathcal{O}(U), \mathcal{G}(U))$ may be interpreted as a worldsheet-like system:

- States correspond to field configurations on a local domain,
- Transformations correspond to gauge or symmetry operations,
- Invariants correspond to physical observables.

In this interpretation, semantic systems behave as dynamical systems defined on local domains, whose global consistency is governed by compatibility conditions across overlaps.

This perspective prepares the transition to a string-theoretic formulation in which global meaning is determined by topological and cohomological constraints.

3 Transition Functions and String Structure

We now interpret the gluing of local semantic regimes in string-theoretic terms.

Let $\{U_i\}$ be an open cover of a domain M. On each U_i, we have a local semantic regime s_i, together with transition data on overlaps

$$g_{ij} : s_i|_{U_i \cap U_j} \to s_j|_{U_i \cap U_j}.$$

These transition functions encode the relation between local interpretations across domains.

3.1 Strings as Transition Data

We interpret each transition function g_{ij} as a string connecting local semantic states.

- The endpoints of the string correspond to local semantic regimes s_i and s_j,
- The string itself represents the transformation between these regimes,
- Composition of transitions corresponds to concatenation of strings.

Thus, strings are not fundamental objects in isolation, but arise as relational structures encoding transformations between local states.

3.2 Gluing and Consistency

On triple overlaps $U_i \cap U_j \cap U_k$, consistency requires:

$$g_{jk} \circ g_{ij} = g_{ik}.$$

This condition expresses the compatibility of strings under concatenation.

When this condition fails, the discrepancy defines an obstruction:

$$g_{jk} \circ g_{ij} = h_{ijk} \cdot g_{ik},$$

where h_{ijk} measures the failure of consistency.

3.3 Interpretation

In this framework:

- Strings encode transitions between local semantic states,
- Gluing corresponds to composition of strings,
- Obstructions correspond to failure of compositional consistency.

Thus, string structure emerges naturally as the geometry of transition data governing semantic descent.

4 Curvature and Obstruction

We now interpret curvature as the failure of consistency of transition data, and hence as a failure of string closure.

4.1 Curvature as Failure of Closure

In the previous section, transition functions g_{ij} were interpreted as strings connecting local semantic regimes. The composition of transitions corresponds to concatenation of strings.

On triple overlaps $U_i \cap U_j \cap U_k$, consistency requires:

$$g_{jk} \circ g_{ij} = g_{ik}.$$

This condition expresses the closure of strings under composition.

When this condition holds, transitions compose coherently and the system admits a consistent global structure.

However, in general, this closure condition may fail:

$$g_{jk} \circ g_{ij} = h_{ijk} \cdot g_{ik},$$

where h_{ijk} represents a deviation from closure.

[Semantic Curvature] Semantic curvature is the obstruction to closure of transition functions, represented by the collection $\{h_{ijk}\}$.

4.2 Cohomological Interpretation

The collection $\{h_{ijk}\}$ defines a Čech 2-cocycle, giving rise to a cohomology class

$$[\{h_{ijk}\}] \in H^2(M, \mathcal{G}),$$

where $\mathcal{G}$ is the sheaf of admissible transformations.

This class measures the failure of transition data to assemble into a coherent global structure.

- Trivial class: transitions close $\rightarrow$ global coherence exists,
- Nontrivial class: transitions fail to close $\rightarrow$ obstruction persists.

4.3 String-Theoretic Interpretation

In string-theoretic terms:

- Strings represent transition data between local states,
- Closed strings correspond to consistent compositions,
- Failure of closure corresponds to nontrivial curvature.

Thus, curvature arises as a geometric property of how strings fail to compose into closed structures.

4.4 Interpretation

Curvature is not an intrinsic property of individual states, but of the relations between them.

- It measures how local transformations fail to assemble globally,
- It encodes the obstruction to coherent semantic structure,
- It manifests as the failure of strings to form closed loops.

In this framework, curvature is precisely the geometric signature of semantic inconsistency.

5 Higher Semantic Structures and Higher-Order Obstructions

The preceding framework identifies curvature as a first-order obstruction to global semantic descent, represented by a cohomology class in

$$H^1(M, \mathcal{G}).$$

We now extend this analysis to higher-order obstructions, which arise from failures of compatibility across multiple overlapping domains.

5.1 Beyond Pairwise Compatibility

The first cohomology group measures incompatibility of transition functions on pairwise overlaps $U_i \cap U_j$.

However, additional structure appears when considering triple overlaps

$$U_i \cap U_j \cap U_k.$$

Even if transition functions satisfy pairwise compatibility, there may exist higher-order inconsistencies in how these compatibilities compose.

[Higher Semantic Obstruction] A higher semantic obstruction is a failure of coherence of transition data on multiple overlaps, represented by a nontrivial cohomology class in

$$H^2(M, \mathcal{G}).$$

Thus, while H^1 measures failure of global sections, H^2 measures failure of consistency of the gluing process itself.

5.2 Gerbes and Higher Gluing Data

The natural geometric object associated with H^2 is a gerbe.

[Semantic Gerbe] A semantic gerbe is a higher-order structure encoding compatibility of transition functions across triple overlaps, rather than merely pairwise overlaps.

In this framework:

- Transition functions g_{ij} define first-order structure,
- Higher data h_{ijk} define compatibility on triple overlaps,
- Failure of trivialization of h_{ijk} corresponds to a nontrivial class in H^2.

Thus, semantic structure is governed not only by transformations, but by higher coherence relations between transformations.

5.3 Stacks and Higher Semantic Regimes

The failure of sheaf conditions at higher levels leads naturally to the notion of stacks.

[Semantic Stack] A semantic stack is a higher categorical structure in which local data, transition functions, and higher coherence relations are all incorporated into a unified framework.

In contrast to sheaves:

- Sheaves encode local data with pairwise gluing,
- Stacks encode local data together with higher-order compatibility conditions.

Thus, stacks provide the natural setting for semantic regimes with nontrivial higher-order obstructions.

5.4 Higher Curvature

The higher obstruction class

$$[\{h_{ijk}\}] \in H^2(M, \mathcal{G})$$

may be interpreted as a higher-order curvature.

[Higher Semantic Curvature] Higher semantic curvature is the obstruction to coherent gluing of transition functions across triple overlaps, represented by a nontrivial class in $H^2(M, \mathcal{G})$.

This extends the notion of curvature from failure of path independence (first order) to failure of coherence of transformation composition (second order).

5.5 Interpretation in String-Theoretic Terms

This hierarchy admits a direct interpretation in string theory:

- H^1 corresponds to gauge fields and holonomy,
- H^2 corresponds to higher gauge fields (e.g., B-fields),
- Gerbes correspond to higher bundles in string theory,
- Stacks correspond to spaces with intrinsic higher-order structure.

Thus, semantic systems naturally organize into a hierarchy of structures analogous to those appearing in string-theoretic models.

5.6 Interpretation

Semantic structure is inherently hierarchical.

First-order obstructions prevent global agreement of local meanings. Higher-order obstructions prevent consistency of the rules governing such agreement.

Thus, meaning is not merely localized, but stratified across levels of coherence.

This suggests that the correct mathematical setting for semantic systems is not that of sets or sheaves alone, but of higher categorical structures in which multiple layers of compatibility are explicitly encoded.

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