Einstein Equations as Global Constraint Laws: A Functorial and Compiler-Theoretic Formulation of Curvature

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Abstract

We present a structural formulation of the Einstein field equations as global constraint laws governing the descent of local geometric data into coherent spacetime structure. Rather than treating curvature solely as a differential quantity, we interpret it as a functorial obstruction to global consistency arising from local-to-global composition.

In this framework, tangent spaces provide local data, coordinate transformations define transition relations, and curvature measures the failure of these structures to assemble into a globally flat geometry. The Einstein equations arise as the unique constraints ensuring compatibility between geometric structure and physical sources, thereby defining the admissible class of spacetime configurations.

This perspective yields a compiler-theoretic interpretation of geometry, in which spacetime is constructed as the output of a transformation system enforcing invariant structure across observational frames. Curvature is thus recast as a constructive invariant governing global coherence rather than a defect of local structure. The resulting framework unifies geometric, physical, and computational viewpoints, enabling the treatment of physical law as an executable system.

Contents

1 Introduction

Modern computational systems suffer from fragmentation between local correctness and global consistency. In contrast, physical theories such as general relativity enforce global structure through local laws. This paper proposes that the Einstein field equations can be understood as structural constraints ensuring that local geometric data descends to a globally coherent spacetime.

The objective is not reinterpretation for its own sake, but the construction of a framework in which physical law can be expressed as an executable transformation system.

2 Structural Framework

We model spacetime as a system defined by local geometric data and their compatibility relations.

- Local data S_i: tangent spaces $T_x M$, each locally isomorphic to R^n
- Transition relations C_{ij}: smooth coordinate transformations on overlapping regions
- Global structure: a differentiable manifold equipped with a metric tensor $g_{\mu\nu}$
- Curvature: the obstruction to assembling locally flat data into a globally flat geometry

Each tangent space admits a flat structure, but global consistency requires that transition maps compose coherently across overlaps. When this compatibility fails, curvature arises.

Thus, curvature measures the deviation between local realizability and global coherence. It is the invariant quantity detecting whether local geometric data descends to a consistent global structure.

3 Einstein Field Equations as Constraint Laws

The Einstein field equations are given by

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

We interpret these equations as global constraint laws governing the admissibility of spacetime geometry.

Structural Interpretation

Let $\{S_i\}$ denote local geometric data (tangent spaces with local coordinate structure), and let $\{C_{ij}\}$ denote transition relations between overlapping regions. A global spacetime $(M, g_{\mu\nu})$ is admissible if and only if these data assemble into a coherent geometric structure satisfying the Einstein equations.

Constraint Principle

The Einstein equations impose a compatibility condition between:

- geometric structure, encoded by $G_{\mu\nu}$
- physical content, encoded by $T_{\mu\nu}$

This condition restricts the class of admissible metrics $g_{\mu\nu}$ to those for which curvature and matter are globally consistent.

Admissibility Statement

A spacetime geometry $(M, g_{\mu\nu})$ is admissible if and only if:

- local geometric data composes coherently across all regions, and
- the resulting global structure satisfies $G_{\mu\nu} = 8\pi T_{\mu\nu}$

Formalization

Definition. A geometric system $(\{S_i\}, \{C_{ij}\})$ is said to be *admissible* if there exists a global structure $(M, g_{\mu\nu})$ such that:

- the local data S_i descends to $(M, g_{\mu\nu})$ under the transition relations C_{ij}, and
- the metric $g_{\mu\nu}$ satisfies the Einstein field equations.

Proposition. A global spacetime $(M, g_{\mu\nu})$ exists if and only if the corresponding geometric system is admissible.

Interpretation. The Einstein equations act as necessary and sufficient conditions for the existence of globally consistent spacetime geometry arising from local data.

Interpretive Consequence

In this formulation, spacetime is not constructed freely, but selected from a constrained space of geometries defined by invariant consistency conditions. The Einstein equations therefore determine the boundary of admissible global structure.

This perspective does not alter physical law, but reveals a structural role for the Einstein equations as governing principles of admissibility and global coherence.

Bridge to Transformation Systems

The admissibility condition defined above can be interpreted operationally as a transformation from local geometric data to global structure under constraint.

Given input data $\{S_i, C_{ij}\}$, the Einstein equations determine whether a corresponding global spacetime $(M, g_{\mu\nu})$ exists. This defines a constraint-driven mapping:

$$(\{S_i\}, \{C_{ij}\}) \longmapsto (M, g_{\mu\nu})$$

whenever admissibility conditions are satisfied.

This mapping provides the conceptual bridge to the compiler interpretation developed in the following section, in which spacetime is understood as the output of a system enforcing invariant global consistency across local data.

4 Functorial Interpretation

The passage from local geometric data to global spacetime structure can be expressed as a structural mapping between systems.

Let $\mathcal{L}$ denote the collection of local frames and coordinate patches, and $\mathcal{G}$ the collection of admissible global geometries. The construction of spacetime defines a mapping

$$F : \mathcal{L} \to \mathcal{G}$$

which assigns to compatible local data a global geometric structure.

This mapping is governed by composition: local transformations must agree on overlaps and compose consistently across chains of regions. When this compositional consistency holds, local data descends to a well-defined global object.

Curvature arises precisely when this compositional structure fails to close. That is, when the induced mappings across overlapping regions cannot be made globally consistent, the result is a nontrivial geometric obstruction.

In this sense, curvature is not an additional structure imposed on spacetime, but the intrinsic signal that local-to-global composition is nontrivial. It reflects the fact that the mapping F cannot be reduced to a globally flat realization.

Thus, spacetime construction can be understood as a composition-driven process, in which global structure emerges from the requirement that local transformations assemble coherently.

5 Compiler Interpretation

The admissibility mapping defined in the previous section can be interpreted as a constraint-driven transformation from local geometric data to global structure.

Given input data $(\{S_i\}, \{C_{ij}\})$, the objective is to determine whether there exists a global spacetime $(M, g_{\mu\nu})$ satisfying the admissibility conditions. This defines a transformation governed entirely by constraint satisfaction:

- Input: local geometric data $\{S_i\}$ and transition relations $\{C_{ij}\}$
- Constraints: compositional consistency and the Einstein field equations
- Output: a global spacetime $(M, g_{\mu\nu})$, when admissibility conditions are satisfied

This process is not generative in the arbitrary sense, but selective: the transformation yields an output if and only if the input data satisfies global consistency constraints.

Thus, the mapping

$$(\{S_i\}, \{C_{ij}\}) \longmapsto (M, g_{\mu\nu})$$

is realized through constraint enforcement. The Einstein equations act as the governing conditions under which this transformation is defined.

From this perspective, spacetime arises as the result of enforcing invariant structure across local data. The transformation is therefore characterized not by construction rules alone, but by admissibility conditions that determine the existence of a valid global realization.

This establishes a compiler-theoretic interpretation in which global geometry is obtained through the evaluation of constraint satisfaction over local structure, with curvature encoding the conditions required for such realization.

Structural Diagram

Input	Constraint Evaluation	Output
$\{S_i\}, \{C_{ij}\}$	$\Longrightarrow$	$(M, g_{\mu\nu})$
Local Geometry	Admissibility Conditions	Global Spacetime

6 Positive Interpretation of Curvature

Curvature is not a defect of geometric structure, but the condition under which nontrivial global geometry becomes possible.

While local regions admit flat representations, global structure is determined by the requirement that these regions assemble coherently. When this assembly is nontrivial, curvature arises as the invariant quantity capturing that structure.

From this perspective, curvature is a constructive feature: it encodes the global relationships that cannot be reduced to local equivalence. It distinguishes geometries that are globally inequivalent despite local similarity.

Thus, curvature should be understood not as a failure of flatness, but as the structural mechanism by which global geometry is realized.

7 Outlook and Applications

The structural formulation developed in this paper suggests a broader computational interpretation of physical law. By viewing the Einstein equations as admissibility constraints over geometric structure, spacetime can be analyzed using methods analogous to constraint evaluation and transformation systems.

This perspective enables the development of formal systems in which local data, compatibility relations, and global consistency conditions are explicitly represented and evaluated. Such systems can be implemented as deterministic compilers that accept structured input and determine the existence and form of admissible global configurations.

Potential applications include:

- geometric verification systems for evaluating admissibility of spacetime configurations
- compiler frameworks for constructing physically consistent models from local data
- formal analysis tools for detecting structural inconsistencies in geometric and physical systems

These directions do not alter physical law, but provide new methods for representing, analyzing, and executing it as a constraint-driven system.

8 Conclusion

We have presented a formulation of the Einstein field equations as global constraint laws governing the descent of local geometric data into coherent spacetime structure. This perspective enables a compiler-theoretic interpretation of geometry, bridging physical law, category theory, and executable systems.

9 References

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