The Einstein Geometry Compiler: A Constraint-Theoretic and Functorial Framework for Curvature and Spacetime Admissibility

Adrian Diamond

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Abstract

We present a compiler-theoretic formulation of general relativity in which spacetime geometry is constructed as the output of a constraint-driven transformation system. Local geometric data, represented by metric tensors and curvature quantities, are treated as input structures subject to global admissibility conditions defined by the Einstein field equations.

Within this framework, curvature is interpreted as an obstruction to global flatness, and admissible spacetimes arise as those configurations satisfying global constraint laws. The Einstein equations function as a compiler that evaluates local geometric data and determines whether a globally consistent spacetime structure exists.

This perspective establishes a deterministic mapping from local metric data and stress-energy content to admissible spacetime geometry, providing a structural interpretation of gravity as constraint-driven geometric computation.

Contents

1 Introduction

General relativity describes gravity as the geometry of spacetime, governed by the Einstein field equations. These equations relate curvature to the distribution of matter and energy.

In this paper, we reinterpret this structure as a constraint-driven transformation system. Local geometric data define input structures, while the Einstein equations act as admissibility conditions determining whether these data assemble into a globally consistent spacetime.

This establishes a compiler-theoretic view in which spacetime geometry is computed as the output of constraint evaluation.

2 Local Geometric Data

Let M be a smooth manifold equipped with a metric tensor:

$$g_{\mu\nu}(x)$$

This defines local geometric structure at each point $x \in M$.

From the metric, one derives curvature quantities such as the Riemann tensor, Ricci tensor, and scalar curvature.

These local quantities constitute the input data of the system.

3 Einstein Equations as Global Constraint

The Einstein field equations are given by:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the stress-energy tensor.

These equations impose global constraints on admissible metric configurations.

Definition. A spacetime metric is admissible if it satisfies the Einstein field equations.

4 Curvature as Obstruction

Flat spacetime corresponds to vanishing curvature. Nonzero curvature represents deviation from global flatness.

Interpretation. Curvature acts as an obstruction to extending local flat structure globally.

Thus, spacetime geometry is determined by the presence or absence of curvature-induced obstruction.

5 Compiler Interpretation

We define a transformation:

$$(\text{local metric data}, T_{\mu\nu}) \longmapsto (\text{admissible spacetime})$$

This transformation is governed by constraint enforcement via the Einstein equations.

- input: local geometric data
- constraint: Einstein equations
- output: admissible spacetime geometry

6 Structural Diagram

Input	Constraint Evaluation	Output
$g_{\mu\nu}, T_{\mu\nu}$	$\Longrightarrow$	Admissible Geometry
Local Geometry	Einstein Constraints	Spacetime Structure

7 Outlook

This formulation enables the construction of systems that evaluate geometric admissibility.

Potential applications include:

- verification of spacetime models
- constraint-based geometry solvers
- computational frameworks for relativistic systems

8 Conclusion

General relativity can be understood as a constraint-driven system in which spacetime geometry arises as the output of admissibility conditions imposed on local geometric data.

This establishes a compiler-theoretic interpretation of gravity as the evaluation of geometric consistency.

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