Quantum Field Theory as an Invariant Compiler: A Functorial and Constraint-Theoretic Formulation of Fields, Symmetry, and Observables

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Abstract

Quantum Field Theory (QFT) is traditionally formulated as a theory of fields, symmetries, and observables defined over spacetime. In this paper, we present a structural reformulation of QFT as a constraint-driven transformation system in which global physical quantities arise as invariant outputs of local field data subject to symmetry and dynamical constraints.

Local field configurations are treated as structured data defined over spacetime, while gauge symmetries act as admissibility conditions governing equivalence classes of configurations. The action functional defines global constraint laws, and the resulting equations of motion determine admissible field evolutions. Observables emerge as quantities invariant under the full symmetry group.

Within this framework, QFT can be interpreted as a deterministic mapping from local field data and symmetry constraints to invariant physical outputs. This establishes a compiler-theoretic perspective in which field configurations are evaluated under admissibility conditions, and physical meaning corresponds to invariant structure under transformation.

Contents

1 Introduction

Quantum Field Theory (QFT) provides the fundamental framework for describing physical systems in terms of fields defined over spacetime. Its structure combines local field dynamics, symmetry principles, and invariant observables into a unified formalism.

Despite its success, QFT is typically presented through analytic techniques and operator formulations that obscure its underlying structural organization.

In this paper, we present a reformulation of QFT as a constraint-driven transformation system. In this view, local field configurations constitute structured input data, symmetry principles define admissibility conditions, and global physical quantities emerge as invariant outputs.

The goal of this work is not to alter the physical content of QFT, but to provide a structural framework in which its components—fields, symmetries, and observables—can be understood as parts of a unified transformation system.

This formulation is equivalent in content to the standard gauge-theoretic description of QFT, but emphasizes structural admissibility, global consistency, and invariant output.

2 Local Field Structure

Let M be a spacetime manifold. A quantum field is defined as a section

$$\phi : M \to E$$

where E is a fiber bundle over M.

At each point $x \in M$, the value $\phi(x)$ represents local field data. The collection of all such values defines a field configuration:

$$\{\phi(x) \mid x \in M\}$$

These local configurations serve as the input data upon which global structure is built.

3 Symmetry as Admissibility Condition

Field configurations are subject to symmetry transformations. In gauge theories, these take the form:

$$\phi(x) \mapsto g(x)\phi(x)$$

where $g(x)$ is an element of a local symmetry group.

Two configurations related by such transformations are physically equivalent. Thus, symmetry defines an admissibility condition.

Definition. A field configuration is admissible up to equivalence under the symmetry group.

The space of physically meaningful configurations is therefore the quotient space under symmetry.

4 Action as Global Constraint Law

The dynamics of a field are governed by an action functional:

$$S[\phi] = \int_M \mathcal{L}(\phi, \partial\phi)\, d^4x$$

Admissible field configurations satisfy the stationary condition:

$$\delta S[\phi] = 0$$

This yields the Euler–Lagrange equations, which act as global constraints on the field.

Thus, the action defines admissibility at the global level.

Proposition: Invariance of Observables

Proposition. Let ϕ be a field configuration and let G be a symmetry group acting on ϕ. Then any observable $\mathcal{O}$ satisfies:

$$\mathcal{O}[\phi] = \mathcal{O}[g \cdot \phi], \quad \forall g \in G$$

Interpretation. Physical observables depend only on equivalence classes of field configurations under symmetry. Thus, the evaluation of observables factors through the quotient space of admissible configurations.

Consequence. The physical output of the system is invariant under admissible transformations, and therefore depends only on structurally consistent configurations.

5 Example: U(1) Gauge Theory

Consider a complex scalar field $\psi : M \to \mathbb{C}$ coupled to a gauge field A_μ.

The local symmetry is given by:

$$\psi(x) \mapsto e^{i\alpha(x)}\psi(x), \quad A_\mu(x) \mapsto A_\mu(x) + \partial_\mu \alpha(x)$$

The associated field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

which is invariant under the gauge transformation.

This defines a $U(1)$ gauge symmetry.

The action is:

$$S[\psi, A] = \int_M \left(|D_\mu \psi|^2 - V(|\psi|) \right) d^4x$$

where the covariant derivative is:

$$D_\mu = \partial_\mu - iA_\mu$$

Structure:

- Local data: $(\psi(x), A_\mu(x))$
- Symmetry: local $U(1)$ transformations
- Constraint: Euler–Lagrange equations
- Output: gauge-invariant observables (e.g., currents, field strengths)

This example demonstrates how local field data, subject to symmetry and dynamical constraints, produces invariant physical quantities.

6 Worked Example: Constraint Evaluation in Gauge Theory

We now describe the full transformation process in a concrete setting.

Step 1: Input (Local Data)

A field configuration consists of:

$$(\psi(x), A_\mu(x))$$

defined over spacetime.

Step 2: Symmetry Reduction

Configurations related by gauge transformations:

$$(\psi, A_\mu) \sim (e^{i\alpha(x)}\psi, A_\mu + \partial_\mu \alpha)$$

are identified as equivalent.
Step 3: Constraint Enforcement
The action functional defines admissibility:

$$\delta S = 0$$

yielding field equations that constrain allowable configurations.
Step 4: Output (Invariant Observables)
Physical quantities such as:

- conserved currents
- field strength $F_{\mu\nu}$
- correlation functions

are invariant under symmetry transformations.
Conclusion.
The system defines a mapping:

$$(\text{local fields}) \longmapsto (\text{invariant observables})$$

subject to admissibility constraints. This realizes the transformation described in Section 10 in a concrete physical setting.

7 Cohomological Interpretation of Gauge Structure

The structure of Quantum Field Theory admits a natural cohomological interpretation in which global consistency of field configurations is governed by descent and obstruction.

Local Data and Transition Relations

Field configurations are defined locally over spacetime:

$$\{\phi(x) \mid x \in M\}$$

Gauge transformations relate local configurations across overlapping regions. These transformations act as transition relations:

$$\phi_i \mapsto g_{ij} \cdot \phi_j$$

on overlaps between local regions.

Descent and Global Structure

A global field configuration exists when local data can be consistently glued together under these transition relations.

This requires compatibility conditions:

$$g_{jk} \circ g_{ij} = g_{ik}$$

ensuring that transformations compose consistently across triple overlaps.

When these conditions are satisfied, the local data descends to a global configuration.

Cohomological Obstruction

Failure of these compatibility conditions produces an obstruction to global consistency.

Definition. A cohomological obstruction arises when locally consistent field data and transition relations fail to assemble into a globally consistent configuration.

In gauge theory, such obstructions correspond to:

- topological defects
- anomalies
- nontrivial field configurations (e.g., instantons, monopoles)

These represent cases where global structure cannot be constructed from local data without residual inconsistency.

Interpretation

From this perspective:

- local fields define sections
- gauge transformations define transition functions
- global configurations arise via descent
- failures of consistency correspond to cohomological classes

Thus, QFT can be understood as a system in which admissibility of field configurations is governed not only by symmetry and dynamics, but by cohomological consistency.

Relation to Constraint Evaluation

The compiler interpretation of Section 10 can now be extended:

- admissible configurations correspond to trivial cohomology classes
- obstructions correspond to nontrivial cohomology
- evaluation consists of determining whether global consistency exists

Thus, the transformation from local data to invariant observables is mediated by cohomological structure, which determines whether admissible global configurations can be realized.

Bridge Principle. The existence of invariant observables depends on the vanishing of cohomological obstruction. Thus, cohomology acts as the structural condition determining whether the transformation from local field data to global observables is well-defined.

8 Example: Quantum Anomaly as Cohomological Obstruction

We now consider an example in which obstruction arises at the quantum level, even when classical consistency conditions are satisfied.

Classical Symmetry vs Quantum Failure

Let a classical field theory possess a continuous symmetry:

$$\phi \mapsto g \cdot \phi$$

Under this symmetry, the action is invariant:

$$S[\phi] = S[g \cdot \phi]$$

and therefore the theory admits a conserved current via Noether's theorem.

However, upon quantization, physical quantities are computed through a path integral:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]}$$

In certain cases, the measure $\mathcal{D}\phi$ fails to remain invariant under the same symmetry transformation.

Anomaly as Obstruction

If under a symmetry transformation we have:

$$\mathcal{D}\phi \neq \mathcal{D}(g \cdot \phi)$$

then the quantum theory no longer preserves the classical symmetry.

This induces a non-invariance of the partition function:

$$Z \neq Z'$$

which implies that the quantum effective action fails to preserve the classical symmetry.

Definition. A quantum anomaly is the failure of a classical symmetry to be preserved after quantization.

This represents a failure of global consistency at the quantum level.

Unified Principle. Physical consistency is equivalent to the absence of cohomological obstruction across both classical and quantum levels.

Cohomological Interpretation

This failure can be interpreted cohomologically:

- local symmetry transformations exist
- classical constraints are satisfied
- quantum corrections introduce a global inconsistency

The anomaly corresponds to a nontrivial cohomology class obstructing symmetry at the quantum level.

Interpretation in the Constraint Framework

Within the framework developed in this paper:

- local data: field configurations ϕ
- symmetry: classical invariance of the action
- attempted global system: quantum theory preserving symmetry
- obstruction: non-invariance of the path integral measure

Conclusion.

A quantum anomaly is a cohomological obstruction that prevents classical symmetry from descending to the quantum theory.

Thus, admissibility at the quantum level requires not only classical consistency, but the absence of cohomological obstruction arising from quantization.

Observation

Observation. Quantum anomalies represent obstructions to symmetry descent induced by quantization, and therefore correspond to nontrivial cohomological structure in the space of field configurations.

9 Invariant Observables

Physical observables are quantities invariant under admissible transformations of field configurations.

Examples include correlation functions, scattering amplitudes, and conserved quantities derived from Noether's theorem.

These observables represent the output of the system:

- Input: local field configurations
- Constraints: symmetry and equations of motion
- Output: invariant physical quantities

Thus, physical meaning corresponds to invariance under admissible transformations.

10 Compiler Interpretation

The structure of QFT defines a transformation:

$$(\{\phi(x)\}, \text{symmetry}, S) \longmapsto \text{observables}$$

This mapping is governed by constraint enforcement:

- symmetry defines equivalence classes
- the action defines admissible dynamics
- observables arise as invariant outputs

The transformation yields output if and only if admissibility conditions are satisfied.

Structural Diagram

Input	Constraint Evaluation	Output
$\{\phi(x)\}, \text{symmetry}, S$	$\Longrightarrow$	Observables
Local Fields	Admissibility Conditions	Invariant Quantities

11 Outlook and Applications

This formulation enables the construction of systems that explicitly evaluate admissibility of field configurations.

Potential applications include:

- verification systems for symmetry-consistent field configurations
- compiler frameworks for constructing invariant physical models
- formal tools for detecting inconsistencies in gauge systems

These directions provide a framework for understanding QFT as a constraint-driven system of invariant computation.

12 Conclusion

Quantum Field Theory can be understood as a constraint-driven transformation system in which local field data, subject to symmetry and dynamical constraints, produces invariant physical observables.

Within this framework, cohomological structure governs the existence of globally consistent configurations, and therefore determines whether invariant physical quantities can be realized.

This perspective unifies fields, symmetry, dynamics, and observables under a single principle: physical meaning arises as invariant structure in the absence of cohomological obstruction.

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